

Stabilizing Chiral Currents in Equilibrium

From Integrable Spin Chains to Cavity Vacuum Realizations

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May 12, 2026

Motivation: The
Quest for
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Transport

General
Theoretical
Framework

Magneto-chiral
States in Spin-1/2
Chains

Generalizing to
Spin-1 Chains

Experimental
Implementation via
Cavity QED

Conclusion &
Outlook

Supported by Armenian Higher Education and Science Committee under research projects 21AG-1C024 and 25Post- Doc1C003.

Outline of the Presentation

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The Fundamental Thermodynamic Constraint

- ▶ **Status Quo:** Conventional quantum systems are reciprocal. In thermal equilibrium, macroscopic currents are strictly forbidden: $\langle J \rangle = 0$.
- ▶ **The Objective:** Realize stable quantum matter that hosts a persistent macroscopic current in the ground state.
- ▶ **Directional Flow:** Breaking reciprocity means the transport properties depend on the propagation direction.
- ▶ **Significance:** Provides a pathway to energy-protected quantum information channels and novel topological orders.

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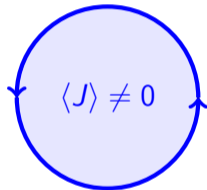
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Intrinsic Non-Reciprocity

- ▶ **Requirement:** Breaking Time-Reversal Symmetry (T) and Parity (P).
- ▶ **Lifting Degeneracy:** Without T -symmetry, forward-moving and backward-moving states are no longer degenerate.
- ▶ This spectral asymmetry is the microscopic engine for macroscopic current emergence.

$$E(k) \neq E(-k) \quad (1)$$



T-Broken Ground State

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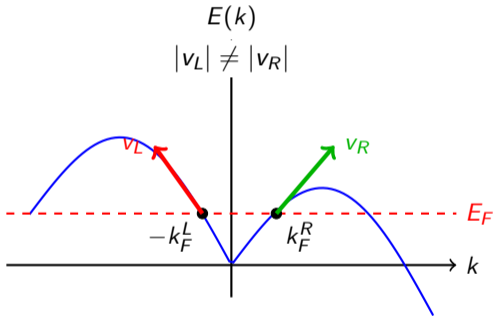
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Non-Reciprocal Excitation Spectra

Tilted Spinon Spectrum

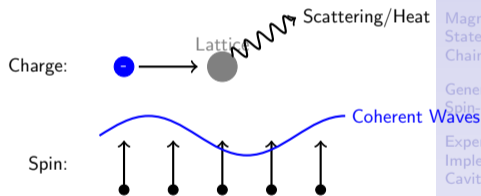
- ▶ Breaking symmetry fundamentally deforms the spinon excitation spectrum.
- ▶ Right-moving and left-moving excitations acquire different group velocities $v_g = \partial E / \partial k$.
- ▶ The Fermi momenta (k_F) become highly asymmetric.
- ▶ This bulk spectral asymmetry allows the system to natively support a macroscopic ground-state current.



The Advantage of Neutral Spin Lattices

Avoiding Electronic Dissipation

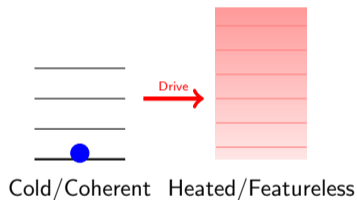
- ▶ Electronic charge transport inherently suffers from Joule heating and phonon scattering.
- ▶ **Spin Lattices:** Magnetic insulators rely on neutral magnetic moments.
- ▶ Information and energy are carried by magnons or spinons without macroscopic charge displacement.
- ▶ **Outcome:** A cleaner platform for studying persistent currents with significantly reduced dissipation.



The Challenge: Driving vs. Heating

The Destruction of Coherence

- ▶ **Floquet Engineering:** Uses strong laser driving to induce currents.
- ▶ **Heating Bottleneck:** Isolated systems lack an energy sink. The drive pumps energy into all many-body modes simultaneously.
- ▶ **Result:** The system heats to a featureless infinite-temperature state, destroying the fragile quantum order we seek to create.



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Stable Chiral Ground States

- ▶ **Motivation:** We must bypass the fundamental heating problem entirely.
- ▶ **Goal:** Engineer stable, zero-temperature ground states that intrinsically carry macroscopic chiral currents.
- ▶ **Methodology:**
 1. Develop a new theoretical framework utilizing deformed integrable models.
 2. Identify a viable experimental platform (Cavity QED) that avoids energy absorption from driving.

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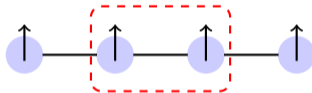
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1D Integrable Spin Lattices

Ideal Platforms for Engineering

- ▶ 1D lattices enhance quantum fluctuations and particle interactions.
- ▶ Integrable models possess an infinite hierarchy of conserved local charges Q_n .
- ▶ These charges commute with the Hamiltonian: $[H_0, Q_n] = 0$.

- ▶ Q_1 : Total Momentum
- ▶ Q_2 : Total Energy (H_0)
- ▶ Q_3 : Energy Current



The Structure of Integrability

- ▶ Integrable systems fundamentally do not thermalize.
- ▶ Conservation laws prevent the system from reaching a standard thermal state.
- ▶ Instead of a Gibbs ensemble, they relax to a Generalized Gibbs Ensemble (GGE):

$$\rho_{GGE} = \frac{1}{Z} \exp \left(- \sum_n \lambda_n Q_n \right) \quad (2)$$

- ▶ Each conserved charge Q_n has its own generalized temperature λ_n .
- ▶ This exact mathematical structure allows us to cleanly deform the system.

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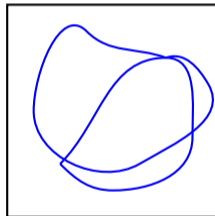
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Constrained Phase Space

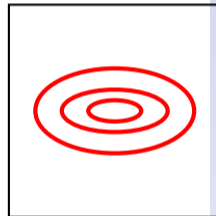
- ▶ Chaotic systems thermalize by ergodically exploring the entire energy shell.
- ▶ Integrable systems are constrained by their infinite conserved charges Q_n .
- ▶ Their trajectories are confined to invariant tori, preventing true thermal mixing.
- ▶ This structure allows us to protect specific high-energy properties mathematically.

Ergodic (Chaotic)



Explores entire shell

Integrable (GGE)



Constrained by Q_n tori

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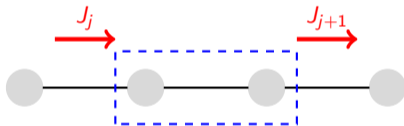
Defining the Energy Current

The Continuity Equation on the Lattice

- ▶ The form of Q_3 is derived from energy conservation.
- ▶ Change in local energy density h_j is balanced by the divergence of the energy current J :

$$i[H_0, h_j] = J_j - J_{j+1} \quad (3)$$

- ▶ Summing J_j over the whole lattice gives the macroscopic conserved current Q_3 .



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The Mathematical Tilt

- ▶ We stabilize a current by introducing a deformation using Q_3 :

$$H(\alpha) = H_0 - \alpha Q_3 \quad (4)$$

- ▶ **Tilt parameter** α : Acts as a chemical potential for the macroscopic current.
- ▶ Because $[H_0, Q_3] = 0$, this deformation does not mix or alter the underlying quantum wavefunctions (eigenstates).
- ▶ It acts to shift the individual energy eigenvalues:

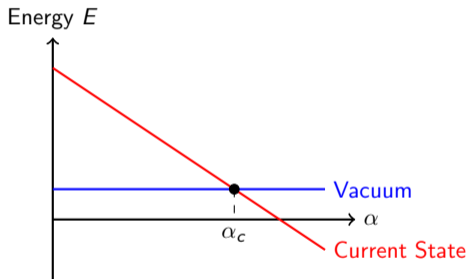
$$E_n(\alpha) = E_n - \alpha q_{3,n} \quad (5)$$

- ▶ This mathematically biases the system toward states carrying a macroscopic flow without changing the physical scattering matrix.

Mapping Excited States to Ground States

The Core Mechanism

- ▶ In H_0 , current-carrying states are highly excited and unstable.
- ▶ The tilt $-\alpha Q_3$ lowers the energy of these specific states linearly.
- ▶ At a critical α , a high-current excited state crosses the original vacuum to become the absolute **ground state** of $H(\alpha)$.



Rigorous Analytical Solutions

- ▶ Integrability allows us to solve $H(\alpha)$ exactly via TBA.
- ▶ The ground state is described by the density of rapidities λ , governed by self-consistent integral equations:

$$\epsilon(\lambda) = \epsilon_0(\lambda) - \alpha q_3(\lambda) + \int_{-\infty}^{\infty} K(\lambda - \mu) \ln \left(1 + e^{-\epsilon(\mu)/T} \right) d\mu \quad (6)$$

- ▶ $\epsilon(\lambda)$: Dressed excitation energy.
- ▶ $\epsilon_0(\lambda)$: Bare excitation energy.
- ▶ $q_3(\lambda)$: Eigenvalue of the energy current operator.
- ▶ $K(\lambda - \mu)$: Two-body scattering kernel.
- ▶ T : System temperature.

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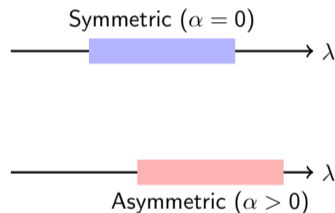
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Macroscopic Current Emergence

- ▶ At $\alpha < \alpha_c$, the Fermi sea of rapidities is symmetric $[-B, B]$.
- ▶ For $\alpha > \alpha_c$, the integration boundaries shift asymmetrically to $[-B_-, B_+]$.
- ▶ This asymmetry directly yields a macroscopic directional flow in the ground state.
- ▶ The transition is determined by the point where a boundary crosses zero energy.



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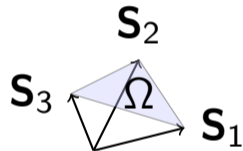
The Scalar Chirality Operator

Definition and Symbols

- ▶ For spin-1/2 lattices, the conserved energy current Q_3 is the scalar chirality operator:

$$Q_3 = \sum_i \mathbf{S}_i \cdot (\mathbf{S}_{i+1} \times \mathbf{S}_{i+2}) \quad (7)$$

- ▶ \mathbf{S}_i : Spin vector operator at lattice site i .
- ▶ \cdot : Scalar (dot) product.
- ▶ \times : Vector (cross) product.
- ▶ Q_3 breaks both Time-Reversal and Parity symmetry.
- ▶ Geometrically represents the solid angle Ω spanned by neighboring spins.



$$\Omega = \text{Solid Angle}$$

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Interplay of Interactions, Chirality, and Fields

- ▶ We consider the most general deformed Spin-1/2 Hamiltonian:

$$H = H_{XXZ} - \alpha Q_3 - hS^z \quad (8)$$

- ▶ H_{XXZ} : Heisenberg Hamiltonian with anisotropy Δ .
- ▶ αQ_3 : Conserved chiral current bias.
- ▶ hS^z : External magnetic (Zeeman) field.
- ▶ This framework allows us to study different physical limits by tuning the anisotropy, the tilt, and the field.

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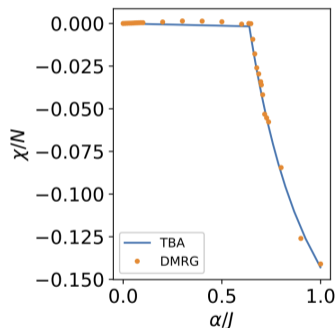
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Limit 1: Chiralization of the XXZ Chain

Response at Zero Magnetic Field

- ▶ Focus on the XXZ model at $h = 0$ (JHEP06(2024)125).
- ▶ Ground state chirality remains zero for small α , protected by the spectral gap.
- ▶ At critical coupling $\alpha_c = 2J/\pi$ (isotropic limit), the gap closes.
- ▶ Transition triggers a persistent macroscopic current in equilibrium.
- ▶ Validates the theoretical paradigm of nonequilibrium features in ground states.



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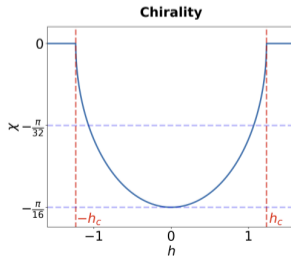
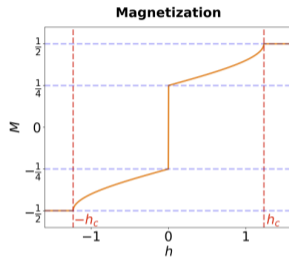
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Limit 2: Magnetochiral States and SSB

Spontaneous Symmetry Breaking

- ▶ Focus on the isotropic Heisenberg chain with finite field (arXiv:2512.09107).
- ▶ Ground state is an atypical magnetochiral XXX eigenstate.
- ▶ **Result:** Spontaneous $SU(2)$ symmetry breaking in 1D.
- ▶ Finite magnetization $M \neq 0$ persists even as $h \rightarrow 0$ in the post-critical sector.
- ▶ Zero-entropy macrostate with ballistic transport.



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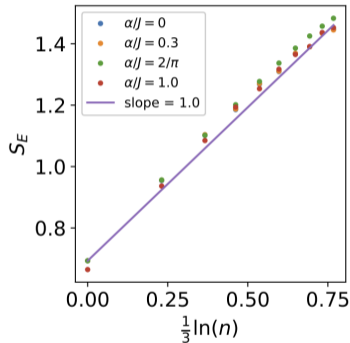
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Universality Class Diagnostics

- ▶ Chiral current-carrying sector is a gapless critical phase.
- ▶ **Central Charge:** $c = 1$ (confirmed for both XXZ and XXX limits).
- ▶ **Finite-Size Scaling:** Probes the stability of the magnetochiral states.
- ▶ Confirms robust symmetry-enriched conformal field theory descriptions.



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Biquadratic Exchange in Spin-1 Lattices

- ▶ We extend the framework to true Spin-1 lattices, which inherently support complex quadrupolar fluctuations.
- ▶ The integrable baseline is the Babujian-Takhtajan (BT) Hamiltonian:

$$H_{BT} = \sum_i \left(\mathbf{S}_i \cdot \mathbf{S}_{i+1} - (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 \right) \quad (9)$$

- ▶ The $(\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2$ biquadratic term differentiates this from simple Heisenberg physics.
- ▶ This model exactly realizes the $SU(2)_2$ Wess-Zumino-Witten conformal field theory and supports the defined energy current charge Q_3 .

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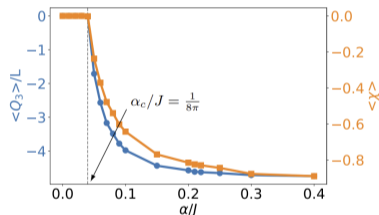
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Dressing via Biquadratic Terms

- ▶ In the BT model, Q_3 is a heavily dressed scalar-chirality operator.
- ▶ Conserved current $\langle Q_3 \rangle / L$ and bare chirality $\langle \chi \rangle$ are distinct.
- ▶ Both turn on at the exact same critical threshold $\alpha_c = J/8\pi$.
- ▶ Results derived from TBA and confirmed via DMRG (arXiv:2603.26897).



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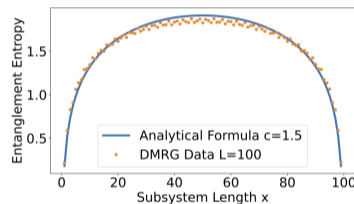
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Competing Orders and $c = 3/2$ Criticality

Intact $SU(2)_2$ WZW Phase

- ▶ Entanglement entropy calculated via DMRG.
- ▶ Data follow the $c = 3/2$ conformal profile.
- ▶ Fractionalized non-Abelian excitations persist in the chiral ground state.
- ▶ Proves that integrability protects the chiral sector from decoherence.



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The Implementation Challenge

- ▶ The theoretical stabilization of currents requires specific three-body interactions (Q_3).
- ▶ Natural solid-state materials typically exhibit only two-body exchange.
- ▶ **Objective:** Use cavity vacuum fluctuations to synthesize these complex terms in equilibrium.
- ▶ Bypasses the heating bottleneck of external Floquet driving.

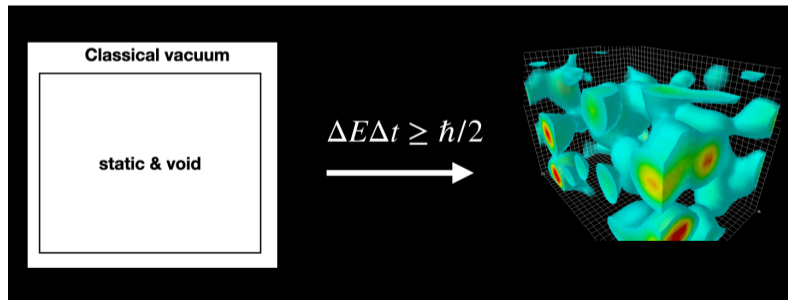
The Quantum Vacuum

Classical Vacuum:

- ▶ Empty space.
- ▶ Static and void.
- ▶ $E = 0, B = 0$.

Quantum Vacuum (QED):

- ▶ Filled with fluctuating fields.
- ▶ Heisenberg Uncertainty:
 $\Delta E \Delta t \geq \hbar/2$.
- ▶ Fields have non-zero expectation values, even at absolute zero temperature.



Visible Vacuum Fluctuations

- ▶ **Spontaneous Emission:** Decays caused by vacuum coupling.
- ▶ **Lamb Shift:** Hydrogen energy shifts from vacuum interaction.
- ▶ **Casimir Effect:** Force between plates due to modified vacuum density.

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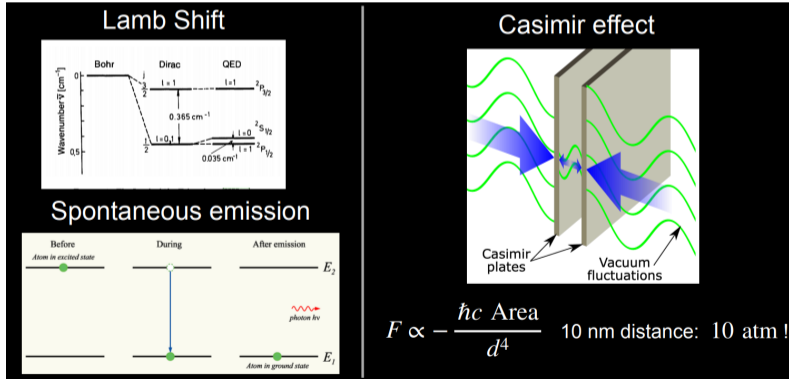
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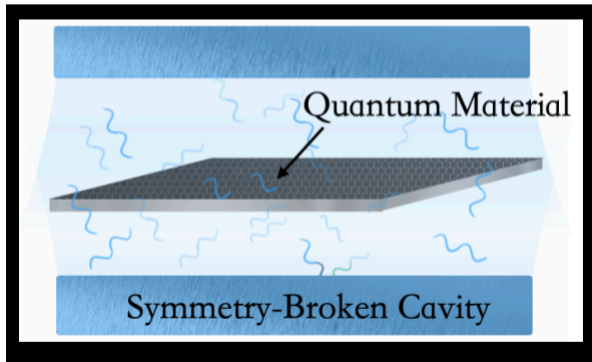
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The Quantum Atmosphere

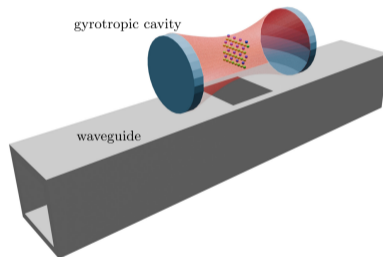
- ▶ Vacuum fluctuations function as a **Quantum Atmosphere**.
- ▶ **Key Insight:** If the boundaries are chiral (gyrotropic cavity), the vacuum fluctuations become chiral.
- ▶ This chiral vacuum imprints its symmetry breaking onto the material inside.



Gyrotropic Cavity Architecture

Engineering the Vacuum Environment (PRL 135, 236901 (2025))

- ▶ Place a material inside a gyrotropic optical cavity.
- ▶ Boundaries break Time-Reversal Symmetry for the photon modes.
- ▶ Vacuum fluctuations acquire a chiral character.
- ▶ This imprints an effective chiral interaction onto the material inside.



Integrating Out Virtual Photons

- ▶ Unitary transform decouples light and matter sectors.
- ▶ The minimal coupling simplifies to a synthetic vector potential \mathcal{A}_{eff} :

$$\mathcal{A}_{eff} = \frac{\tilde{m}\xi^2}{2}\hat{z} \times \nabla V_{lattice}(r) \quad (10)$$

- ▶ The cavity-renormalized electron mass \tilde{m} , characteristic length ξ , and shifted cavity frequency $\tilde{\omega}_c$ are defined by bare physical parameters:

$$\tilde{m} = m + \frac{q^2 A_0^2}{\omega_c}, \quad \xi = \frac{qA_0}{m\tilde{\omega}_c}, \quad \tilde{\omega}_c = \omega_c + \frac{q^2 A_0^2}{m} \quad (11)$$

- ▶ m : Bare mass. q : Charge. A_0 : Vacuum field amplitude. ω_c : Cavity frequency.

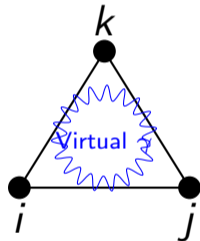
Synthesis of Q_3 via Virtual Photons

Resulting Effective Interaction

- ▶ Virtual exchange of chiral photons generates multi-spin bonds.
- ▶ A t/U expansion of the Hubbard model yields exactly the scalar chirality term:

$$H_{\text{eff}} = H_0 - \kappa \sum_i \mathbf{S}_i \cdot (\mathbf{S}_{i+1} \times \mathbf{S}_{i+2}) \quad (12)$$

- ▶ H_{eff} is exactly the tilted Hamiltonian we study theoretically.
- ▶ Realization occurs in true thermal equilibrium.



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Non-Invasive Cavity Readout

- ▶ The cavity acts as its own non-invasive probe through the transmittance $T_k = |t_k|^2$.
- ▶ Transmission amplitude t_k is determined by the average photon number \bar{N} :
$$t_k = 1 - \frac{1}{2}\omega_k |g_k|^2 - \left(\omega_k + \frac{1}{2}\right) |g_k|^2 \bar{N}$$
- ▶ \bar{N} equals the electronic kinetic energy in the asymptotically decoupled frame: $\bar{N} = \frac{1}{2}\xi^2 \langle p^2 \rangle = m\xi^2 \langle H_{kin} \rangle$
- ▶ By the Hellmann-Feynman theorem, the kinetic energy in the effective spin model is linked to the chiral order Q_3 : $\langle H_{kin} \rangle = 2\langle H_{Heis} \rangle + 3\alpha \langle Q_3 \rangle$

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A Unified Approach

- ▶ **Theoretical Paradigm:** Reconstructed non-equilibrium chiral currents into stable ground states using $H_0 - \alpha Q_3$.
- ▶ **Model Application:** Demonstrated symmetry breaking and gapless CFT phases across Spin-1/2 ($c = 1$) and Spin-1 ($c = 3/2$) lattices.
- ▶ **Experimental Solution:** Gyrotropic Cavity QED naturally synthesizes complex interactions without laser heating.
- ▶ **Future Impact:** Provides a realistic roadmap for engineering robust transport of quantum information in strongly correlated systems.